

# Capacity-Constrained Network-Voronoi Diagram: An Extended Abstract

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## Summary

Given a transportation network with weighted nodes (e.g., population, resource demand) and a set of service centers (e.g., gas stations) with capacity constraints, a Capacity Constrained Network-Voronoi Diagram (CCNVD) partitions the network such that load imbalance across service centers is minimized. A CCNVD is important for resource allotment during natural disasters such as Hurricanes Sandy and Katrina. Creating a CCNVD is challenging due to the uneven geographic distribution of service centers and consumers, which may lead to long wait times for the affected population. Previous work has focused on either discovering an optimal location for one or more service centers (e.g., location-allocation) or minimizing the time the affected population travels to the nearest service center. None has considered the load at the service center. By contrast, we propose a pressure equalization algorithm to create a CCNVD where service center locations are fixed and the load across service centers is balanced. We present a case study showing the effectiveness of our approach in reducing load imbalance as compared to traditional network Voronoi diagrams.

## 1. Introduction

Recent fuel shortages during hurricane Sandy serve as a reminder of the importance of resource allotment amid natural or man-made disasters such as floods, hurricanes, tsunami, fire, terrorist acts, and industrial accidents. Uneven or skewed distribution of resources may lead to chaos, panic, and suffering. During hurricane Sandy, New Jersey residents had to “wait for hours to fill up car tanks and containers for home generators” (ABC News, (2012)). On the other hand, ensuring safe and stable resource allotment may assist authorities in smoothing vehicle (or people) movement during a critical resource shortage situation (e.g., gas rationing) and reduce the risk of congestion and travel time delays. This paper explores the problem of effective resource allotment to supply required resources to users, which we conceptualize as the creation of a Capacity Constrained Network Voronoi Diagram (CCNVD).

Given a transportation network with weighted nodes (e.g., population, resource demand) and a set of service centers (e.g., gas stations) with capacity constraints (e.g., amount of gasoline, size of parking lot), a CCNVD partitions the network and assigns every node to a destination such that load imbalance across service centers is minimized.

This problem is challenging due to the uneven geographic distribution of service centers and consumers, which may lead to uneven loads across service centers, thereby creating unnecessary waiting at overloaded service centers for the affected population. For example, Figure 1(a) shows vehicles waiting in line for fuel at a gas station and

Figure 1(b) shows people waiting line at a gas station, both in the New York-New Jersey area.



(a) Vehicles wait in line for fuel at gas station in the New York-New Jersey area, on Nov. 1, 2012.

(Courtesy: [www.bloomberg.com](http://www.bloomberg.com))



(b) Waiting line at a gas station on Nov. 1, 2012 in the New York-New Jersey area

(Courtesy: Andrew Burton/Getty)

**Figure 1. Nearest destination allotment may lead to load imbalance and resource shortages (e.g., Gas shortage problem after Hurricane Sandy)**

Some early work on resource allotment focused on location-allocation or discovering an optimal location for one or more centers that will service demand from a given set of points (Densham et al. (1991)). The problem at hand, however, is to reduce load imbalance between a given set of service centers based on the affected population, not discover new locations for service centers. A second line of research work has investigated how to minimize the time the affected population travels to the nearest service center by creating Voronoi diagrams. Early Voronoi diagrams (VD) were based on the Euclidean distance metric (as the crow flies) to assist in resource allotment (Aurenhammer (1991), Samet (2006), Balzer et al. (2008), Balzer (2009)). The Voronoi diagrams ignored network distance (e.g., shortest path) and network constrained properties (e.g., direction), which are important in the context of transportation networks. To remedy this limitation, various approaches on Network Voronoi Diagrams (NVD) have been proposed in recent literature (Okabe et al. (2009), Okabe & Sugihara (2012)). In comparison with VDs, NVDs provide more insightful information for understanding transportation networks. However, previous NVDs have not been designed to account for load-imbalance across service centers.

By contrast, this paper proposes a novel approach for creating Capacity Constrained Network Voronoi Diagrams (CCNVD) that handles load imbalance. Our preliminary analysis shows that CCNVD leads to better load balance across service centers on a network relative to previous work.

## 2. Problem Statement

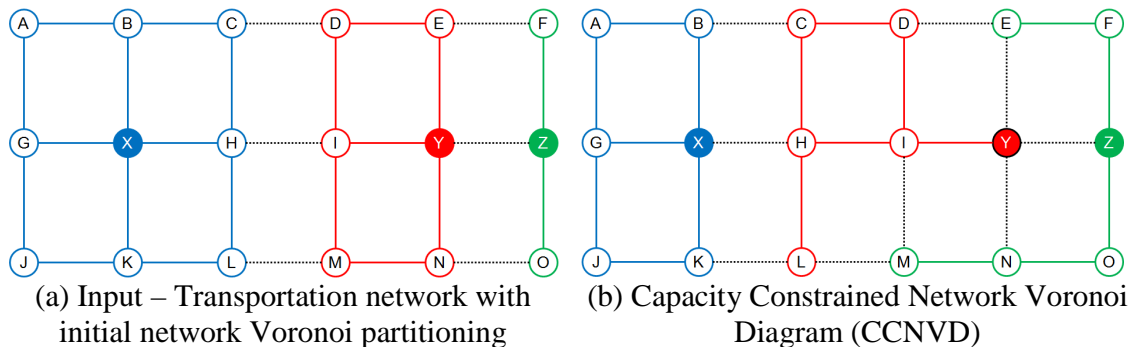
In our problem formulation, a transportation network is represented and analyzed as a graph composed of nodes and edges. Every node represents a spatial location in geographic space (e.g., road intersections). Every edge represents a connection between two nodes and has a travel time and direction. Nodes have an initial occupancy (e.g., population or demand) and destinations (e.g., service centers) have a capacity for people (supply). The CCNVD problem may be formalized as follows:

**Given:** A transportation network with nodes  $N$  and edges  $E$ , the total number of consumers and their locations, the locations and capacity of all destinations (e.g., service centers), and an initial network Voronoi partitioning.

**Output:** Capacity Constrained Network Voronoi Diagram showing destination area for each node.

**Objective:** Minimize load imbalance across destinations.

**Constraints:** Every node has a single destination.



**Figure 2. Example Input and Output of Capacity Constrained Network Voronoi Diagram**

Figure 2 illustrates the input and output of CCNVD with a transportation network (18 nodes with 15 consumer nodes (A,B,...,O) and three service center nodes (X, Y, and Z)). For simplicity, we assume that every edge has the same travel time (i.e., unit travel time) and every consumer node has unit consumer demand. Figure 2(a) shows a Network Voronoi Diagram (NVD), allotting each node to the nearest destination (e.g., shortest path). NVD assigns 8 consumer nodes (color=blue) to service center X, 5 consumer nodes (color=red) to service center Y, and 2 consumer nodes (color=green) to service center Z. The dotted lines represent the boundary between adjacent Voronoi regions. Although NVD allots each node to the nearest service station, it does not account for load balance, which may lead to congestion and delay at service station X while service station Z is left with few consumers. Figure 2(b) shows an example of the proposed Capacity Constrained Network Voronoi Diagram (CCNVD). As can be seen, the load is balanced and 5 nodes are allotted to each service station as shown by distinct colors.

### 3. Proposed Algorithm

The Pressure Equalization (PE) algorithm aims to reduce load imbalance across service stations when creating a Capacity Constrained Network Voronoi Diagram (CCNVD). It starts with the partitions of a Network Voronoi Diagram and iteratively adjusts the distribution of allotted nodes until the load is balanced across all service station destinations. A core idea in PE is the Pressure Equalization Directed Acyclic Graph (PEDAG). The nodes of PEDAG are all the destinations in the transportation network, where each destination  $d$  has a current allotment of nodes. If  $d$ 's current allotment exceeds its capacity,  $d$  is called an *excess destination* and the amount by which the capacity is exceeded is called the *excess*. On the other hand, if  $d$ 's current allotment is below its capacity,  $d$  is called a *deficit destination*, with the difference between the

current allotment and capacity being the *deficit*. A directed edge from PEDAG node  $d$  to PEDAG node  $d_2$  is inserted if  $d$ 's allotment is greater than  $d_2$ 's allotment. Figure 3(b) shows the PEDAG for the network Voronoi diagram of Figure 2(a) (reproduced in Figure 3(a)). It has three nodes for service stations X, Y, and Z. Node X has an excess capacity of 3 and node Z has a deficit of 3. There are two edges showing that the service area of Y is adjacent to the service area of X and Z. Arrow direction ( $X \rightarrow Y$ ,  $Y \rightarrow Z$ ) shows comparison of load.

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**Algorithm 1** Pressure Equalizer (PE)

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**Inputs:**

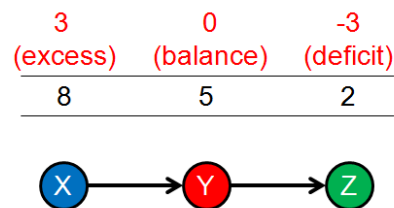
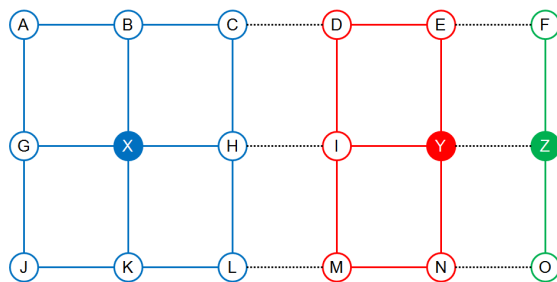
A transportation network with nodes ( $N$ ) and edges ( $E$ ), capacity on destinations, travel time on edges, population on nodes.

**Output:** Capacity Constrained Network Voronoi Diagram (CCNVD)

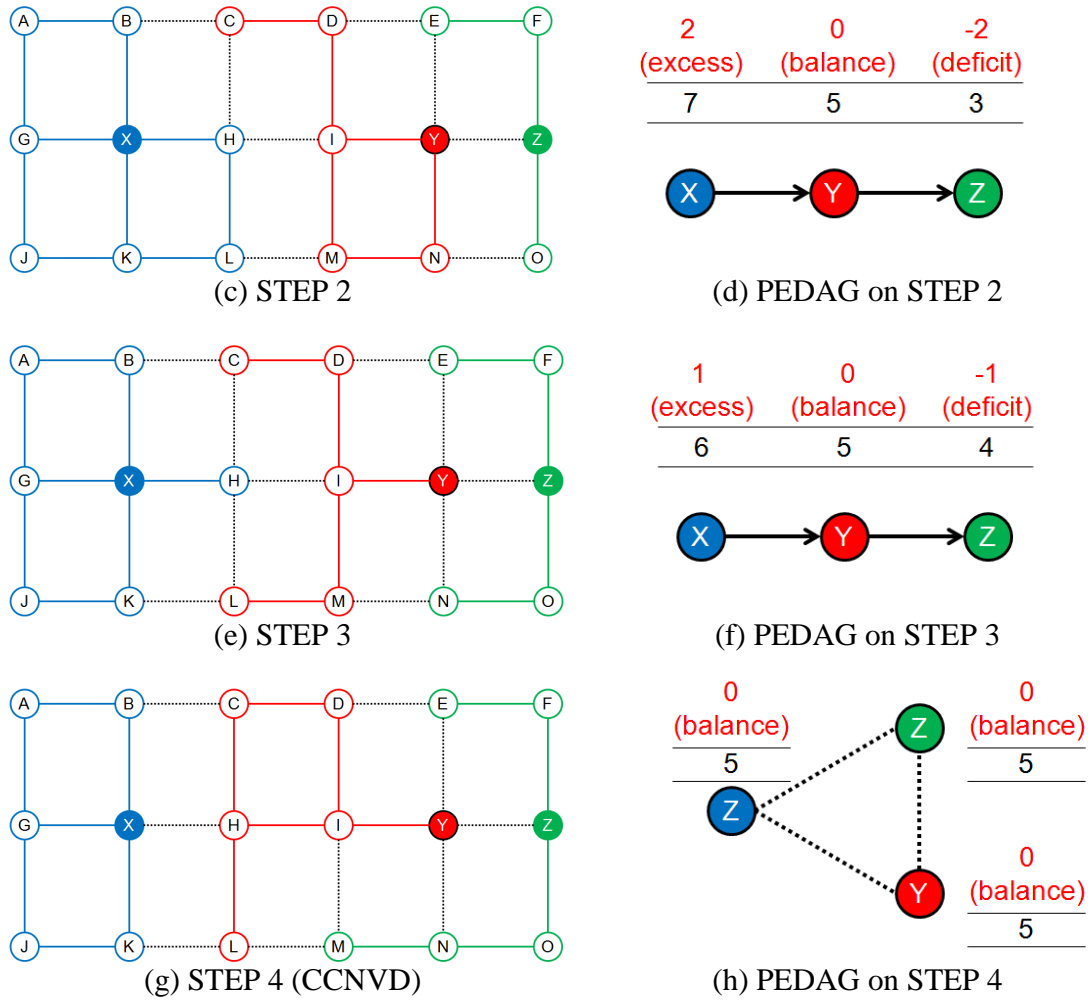
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- 1: CCNVD  $\leftarrow$  NVD (Network Voronoi Diagram)
  - 2: Construct Pressure Equalization directed acyclic graph (PEDAG) using NVD.
  - 3: Find root and leaf destinations on PEDAG.
  4. **while** [ |Avg. excess(root destination)| + |Avg. deficit(leaf destination)|  $\geq$  1 ]
  5. Choose a (root( $u$ ), leaf( $v$ )) pair, s.t. there is a path ( $p$ ) from  $u$  to the  $v$ .
  6. Move one consumer node from root  $u$  to leaf  $v$  via path  $p$ .
  7. Update CCNVD to reflect new allotment.
  8. Update PEDAG and identify new roots and leaves.
  - 9: **end while**
  - 10: return CCNVD
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Algorithm 1 presents the pseudocode for PE. First, PE initializes the CCNVD with the given NVD (Line 1). It then creates a PEDAG and identifies the roots and leaf destinations on it (Lines 2-3). Then PE iteratively reallocates nodes from selected root  $u$  to selected leaf  $v$  until the allotment is in line with service station capacity (Lines 4-9). During this re-allotment, every service station picks the nearest consumer node in the service area (Line 6). Finally, the updated CCNVD with balanced destinations is returned (Line 10).



(b) PEDAG on STEP 1



**Figure 3. Example of PE algorithm**

Figure 3 shows the execution of the proposed PE algorithm (Algorithm 1) on the network Voronoi diagram (NVD) input shown in Figure 2. PE starts with NVD as an initial solution (Figure 3(a)) and creates a PEDAG (Figure 3(b)). In this example, the destination with highest excess is C because its allotted nodes are greater than those of other destinations (Figure 3(b)). Next, PE re-allots a node from X to Z, via Y (Figure 3(c)) and updates the PEDAG (Figure 3 (d)). Note that every station picks the nearest consumer node in service area (service station Y picks node C and service station Z picks node E). After several iterations, PE achieves a balanced allotment (Figure (h)) and the algorithm terminates. Figure 3(g) shows the resulting Capacity Constrained Network Voronoi Diagram.

**Analysis:** The Pressure Equalization (PE) algorithm terminates on all inputs. During each iteration, the algorithm chooses a root destination and re-allots a node through a path  $p$  on the PEDAG. Intuitively, the average excess across roots decreases by one and the average deficit across leaves increases by one. Then one iteration of PE reduces the sum of the average excess across a root and the absolute value of the average



deficit across a leaf. After at most  $n$  iterations, the sum should be less than 1 when the algorithm terminates.

We plan to characterize the computational complexity of the PE algorithm in future work. In the case study (Section 4) with 7,407 consumer nodes and 9 service stations, the algorithm completed in 900 milliseconds on an Intel Core i7-2670QM CPU machine running MS Windows 7 with 8 GB of RAM.

#### 4. Case Study

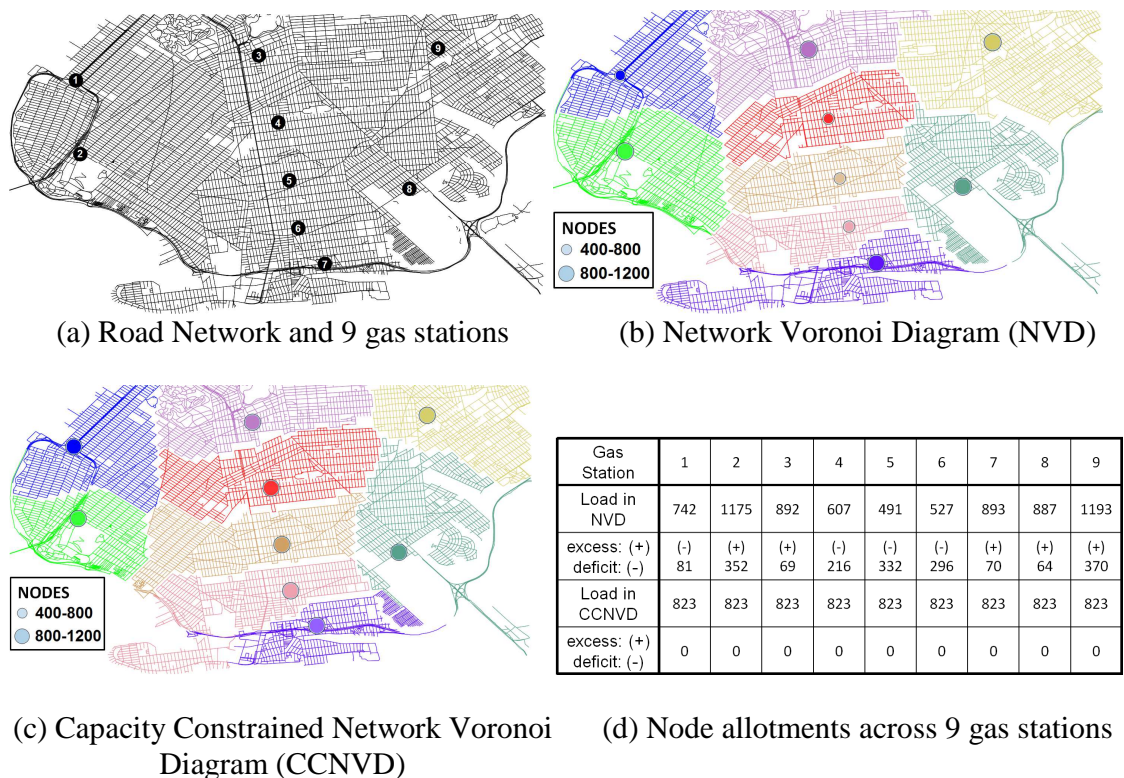


Figure 4. Case Study of gas station allotments on Brooklyn

Our experiments used a Brooklyn, NY road map consisting of 7,407 nodes and 24,980 edges, taken from OpenStreetMap (Figure 4(a)). We chose 9 gas stations and created a Network Voronoi Diagram (NVD), as shown in Figure 4(b). For simplicity, we assume that all gas station have some capacity and the gas station together can serve all people allowed on a specific day of week. The number of allotted nodes for every gas station is represented by the size of circles. In our analysis, the NVD shows unbalanced allotments, which may lead to longer wait time for larger regions. Figure 4(c) shows the CCNVD produced by our algorithm. Table in Figure 4(d) shows the load on every service station in NVD and CCNVD. It shows that CCNVD is able to remove excesses and deficits to balance load.

As can be seen, all destinations are balanced in terms of the number of allotted nodes. For example, the green gas station (gas station 2) in the NVD services 1,175 nodes and the

brown gas station (gas station 5) to its right services 491 nodes, which is an imbalance. In the corresponding CCNVD, both green and brown gas stations service 823 nodes. Our preliminary analysis shows that CCNVD has more balanced and stable allotment compared to NVD.

## 5. Conclusion

We presented the problem of creating a Capacity Constrained Network Voronoi Diagram (CCNVD), which is important for resource allotment during natural and technological disasters (e.g., Hurricane Sandy). Creating a CCNVD is challenging due to the uneven geographic distribution of service centers and consumers, which may lead to long wait times for the affected population. In this paper, we introduced a novel approach, namely the Pressure Equalization (PE) algorithm, to create a CCNVD that minimizes load imbalance across service centers. We presented a case study showing that CCVND resulted in less load imbalance than traditional network Voronoi diagrams.

## 6. Acknowledgements

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